Diophantin solutions for $a^3+b^3+c^3=d^5+1$

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In this paper we consider Diophantine equations of the type $a^3 + b^3 + c^3 = d^5 + 1$, as well as related equations $a^3 + b^3 + c^3 = d^5$ and $a^3 + b^3 = c^3$. Examples of solutions are given. We give parametric solutions.

I. INTRODUCTION

A few examples of solutions to

$$a^3 + b^3 + c^3 = d^5 + 1 \tag{1}$$

$$\begin{aligned}
&6^3 + 3^3 + 1^3 = 3^5 + 1,\\
&8^3 + 8^3 + 1^3 = 3^5 + 1,\\
&40^3 + 33^3 + 4^3 = 10^5 + 1,\\
&53^3 + 23^3 + 2^3 = 11^5 + 1.
\end{aligned}$$

II. MAIN RESULTS

A. Trivial solutions

Some trivial solutions:

$$a = b = 2^{5k+3},\ c = 1,\ d = 2^{3k+2} \tag{2}$$

$$a = b = 2^3k^5,\ c = 1,\ d = 2^2k^3 \tag{3}$$

This parametrization shows that there are an infinite number of solutions to the (1).

B. Equation of the type $a^3 + b^3 + c^3 = d^5$

A few examples of solutions to

$$a^3 + b^3 + c^3 = d^5 \tag{4}$$

$$\begin{aligned}
&18^3 + 12^3 + 6^3 = 6^5,\\
&27^3 + 27^3 + 27^3 = 9^5,\\
&60^3 + 32^3 + 4^3 = 12^5,\\
&60^3 + 30^3 + 18^3 = 12^5.
\end{aligned}$$

Parametric solutions

Every solution of

$$a^3 + b^3 + c^3 = d^5$$

generates a family of solutions

$$(at^5)^3 + (bt^5)^3 + (ct^5)^3 = (dt^3)^5, \tag{5}$$

where $t = 1, 2, 3, ...$
Another parametrization:

$$\begin{aligned} a &= m(m^3 + n^3 + p^3)^3 \\ b &= n(m^3 + n^3 + p^3)^3 \\ c &= p(m^3 + n^3 + p^3)^3 \\ d &= (m^3 + n^3 + p^3)^2 \end{aligned} \tag{6}$$

$$\tag{7}$$

Parametrization assures that there is an infinite number of solutions to (4).

C. Equation of the type $a^3 + b^3 = c^3$

A few examples of solutions to

$$a^3 + b^3 = c^5 \tag{8}$$

$$\begin{aligned} &6^3 + 3^3 = 3^5, \\ &8^3 + 8^3 = 4^5, \\ &192^3 + 96^3 = 24^5, \\ &1488^3 + 729^3 = 81^5. \end{aligned}$$

Here, according to Beal's conjecture, a, b, c have common factors.

Parametric solutions

Every solution of

$$a^3 + b^3 = c^5$$

generates a family of solutions

$$(at^5)^3 + (bt^5)^3 = (ct^3)^5, \tag{9}$$

where $t = 1, 2, 3, ...$
Another parametrization:

$$\begin{aligned} a &= m(m^3 + n^3)^3 \\ b &= n(m^3 + n^3)^3 \\ c &= (m^3 + n^3)^2 \end{aligned} \tag{10}$$

Parametrization assures that there is an infinite number of solutions to (8).

III. CONCLUSIONS

In this work we studied Diophantine equations of the type $a^3 + b^3 + c^3 = d^5 + 1$, as well as related equations $a^3 + b^3 + c^3 = d^5$ and $a^3 + b^3 = c^3$. Examples of solutions, as well as parametric solutions were given.

Thanks who helped...

References:

[1] ???